THE APRIL MEETING OF THE SAN FRANCISCO SECTION

The thirty-ninth regular meeting of the San Francisco Section of the American Mathematical Society was held at Stanford University on Saturday, April 8. Professor R. E. Allardice presided. The total attendance was twenty-one, including the following sixteen members of the Society:

Alderton, Allardice, Bernstein, Blichfeldt, Buck, Cajori, Daus, Haskell, Hoskins, Lehmer, Libby, W. A. Manning, Moreno, F. R. Morris, Noble, Pauline Sperry.

The Secretary of the Section was directed to send a letter of felicitation to the Chicago Section on the occasion of its twenty-fifth anniversary, and a letter of felicitation to Professor Scott on the occasion of the celebration held in her honor at Bryn Mawr College on April 18.

The dates of the next two regular meetings of the Section were fixed as October 21, 1922, University of California, and April 7, 1923, Stanford University.

Titles and abstracts of papers read at this meeting follow below. The papers of Professors Bell and Carpenter were read by title.

1. Dr. P. H. Daus: *On the solution of the diophantine equation* $x^3 - Dy^3 = \pm 1$. Preliminary report.

The author finds solutions of the equation $X^3 - DY^3 = \pm 1$ from the solutions of the pellian cubic $x^3 + Dy^3 + D^2z^3 - 3Dxyz = 1$, and shows by means of ternary continued fractions that the first equation cannot have more than one non-trivial solution, and if it has a solution, it is obtainable from the fundamental solution of the pellian cubic.

2. Professor Florian Cajori: *Recent symbolisms for decimal fractions.*

The author describes the struggle for supremacy of the dot and the comma in different countries during the eighteenth and nineteenth centuries, as the decimal separatrix. During the past thirty-five years over half a dozen different decimal

notations have appeared in print, viz., 2.5 (in the United States), $2 \cdot 5$ (in Great Britain), 2,5 (the common notation on the European Continent), $2'5$, $2'5$, 2,5 (three notations occasionally found in Spanish books), $2_{,5}$, $2._5$ (two notations found in Scandinavia and Denmark).

3. Professor Florian Cajori: *Pricked letters and ultimate ratios.*

In the PHILOSOPHICAL TRANSACTIONS (London), vol. 3, p. 763, N. Mercator used in 1668 the dot placed over the letter I or over a number, to indicate an infinitesimal difference, similar to the use of the dot in $\dot{x}$ among certain English writers (after Newton in 1693 had used $\dot{x}$ in print to designate a fluxion or velocity) to denote infinitely small quantities. Mercator used in 1668 the Latin forms for "prime" and "ultimate little ratios"; Newton first used in print the phrase "prime and ultimate ratios" in 1687. It is possible that Newton adopted this terminology from Mercator, but his use of the dot in private papers antedates Mercator's use of it in print.

4. Professor B. A. Bernstein: *The complete existential theory of Hurwitz's postulates for abelian groups and fields.*

In a previous paper, the author determined those sets of Hurwitz's postulates for abelian groups and fields that are completely independent. In the present paper he obtains the complete existential theory of Hurwitz's remaining postulate-sets and thus establishes the complete existential theory of all of the sets. The complete paper will appear in this BULLETIN.

5. Professor E. T. Bell: *Periodicities in the theory of partitions.*

Theta functions, not merely theta constants as customary, being used in the analysis of partitions, it is shown that Euler's $P(n)$ is connected with six new denumerants by a remarkable system of periodic relations. Two kinds of periodicity are involved: ordinary (as in analysis); numerical (periodicity of residues with respect to an integer modulus). Either or both can occur in one relation. The theory is closely connected with cyclotomy, Lucas' u_n, v_n, and the algebraic integers of a realm. The paper will appear in the ANNALS OF MATHEMATICS.

6. Professor E. T. Bell: *A revision of the Bernoullian and Eulerian functions.*

Four functions of a single variable are taken as fundamental instead of the usual pairs of Raabe, Schlömilch, Worpitzky, Glaisher and others, and by these the theory is reduced by the symbolic method to a simple and symmetric form.

7. Professor E. T. Bell: *Trigonometry and the numbers B, E, G, R of Bernoulli, Euler, Genocchi, and Lucas.*

All general relations between numbers B, E, G, R of different ranks can be obtained briefly and systematically by Blissard's umbral calculus as an isomorph of trigonometry. A set of 22 such relations involving an arbitrary analytic function is given. The set is complete in that it contains at least one relation of each type. A relation is of type r if it involves precisely r of the symbols B, E, G, R.

8. Professor C. A. Noble: *Retention of a salt solution in a tank of flowing water.*

The author assumes that a salt solution of given strength is continuously admitted into a stream of water which flows uniformly through a system of n tanks; that the contents of each tank is agitated; and that the salt diffuses instantaneously. He deduces the differential equation for the amount of salt remaining in the nth tank at the time t, and solves this equation.

9. Professor A. F. Carpenter: *Generalization of a theorem on flecnode curves of ruled surfaces.*

In this paper the author proves that a necessary and sufficient condition that the planes osculating the flecnode curve and the complex curve of a ruled surface at the four points in which they intersect any ruling form a harmonic pencil is the vanishing of the invariant

$$\frac{p_{21}}{p_{12}}\Delta_1 - \frac{p_{12}}{p_{21}}\Delta_2.$$

This theorem generalizes certain results formerly obtained by the author (TRANSACTIONS OF THIS SOCIETY, vol. 16 (1915), p. 509). The paper further determines the differential equations of the ruled surface generated by the axis of the pencil of osculating planes and shows that if the tangents to one branch of the flecnode curve of the given ruled surface

cut out one branch of the flecnode curve of the second ruled surface, the same is true of the tangents to the other branch of the flecnode curve, that a corresponding situation holds for the respective complex curves, and finally that the two ruled surfaces are under these conditions space duals.

10. Professor Florian Cajori: *The history of trigonometric notations.*

The author describes trigonometric notations used by early writers, beginning with Thomas Finck, who in 1583 used contractions *sin., tan., sec., sin.com., tan.com., sec.com.;* he had very few followers in his time. In Great Britain, in 1618 and 1631, severe contractions like *s, t, s co, t co* were introduced by Oughtred, Norwood and others for sine, tangent, cosine, cotangent, but seldom did any two authors or even any one author always adhere to exactly the same set of symbols. About 1760 these severe contractions were abandoned in England. There and on the continent abbreviations of three or four letters (five letters for *cosec*) were usually preferred, but no unanimity exists even today. The origin is traced of notations for the inverse functions (arc sin (Euler), $\sin^{-1}x$ (J. F. W. Herschel), $\frac{1}{\sin}x$ (M. Ohm)), also for hyperbolic functions (sinh (V. Riccati and J. H. Lambert), Sin (C. Gudermann)) and for parabolic functions (J. Booth).

11. Dr. F. R. Morris: *Arrangement of interest tables for Liberty Bonds.*

The usual plan of interest tables for bonds is to place the time until maturity and rate of income in the two margins and the value of the bond in the body of the table. However, it is often desirable to read the rate directly. In the case of Liberty Bonds, compact tables may be constructed such that rate may be read if the market price on a given date is known. With time and rate as arguments the computation is direct; with time and value as arguments the computation is indirect and rather difficult.

12. Professor H. F. Blichfeldt: *Notes on diophantine approximations.* Preliminary report.

The following is a well known theorem: n irrational numbers $\alpha_1, \cdots, \alpha_n$ being given, as well as a positive number ϵ ar-

bitrarily small, then a set of n rational fractions $x_1/y, \cdots, x_n/y$ can be found such that $|\alpha_i - x_i/y| < \epsilon$ and also $< \delta/y^{1+1/n}$ $(i = 1, 2, \cdots, n)$, where δ is a fixed number depending upon n only (cf. TRANSACTIONS OF THIS SOCIETY, vol. 15 (1914), pp. 234–5; L. E. Dickson, *History of the Theory of Numbers*, vol. 2, p. 95 ff.). A. Hurwitz has proved (loc. cit.) that $\delta = 1/\sqrt{5}$ for $n = 2$; and H. F. Blichfeldt that $\delta \geqq 1/\sqrt[3]{23}$ for $n = 3$ (proof not yet published). In MATHEMATISCHE ANNALEN, vol. 83 (1921), p. 77 ff., there is a proof by O. Perron that $\delta > (1/n)[0.35/(n+1)]^n$ approximately. The present writer, using Perron's method with refinements based on the principles of the geometry of numbers, obtains the somewhat better result $\delta > (N/\sqrt{n})(2/\sqrt{n})^n$ approximately, where N is a fixed number.

B. A. BERNSTEIN,
Secretary of the Section.

THE EASTER MEETING OF THE SOCIETY

The two hundred twenty-second regular meeting of the American Mathematical Society, being the seventeenth regular Western meeting, and the forty-ninth regular meeting of the Chicago Section, was held at the University of Chicago on Friday and Saturday, April 14 and 15, 1922, in honor of the twenty-fifth anniversary of the Chicago Section. The attendance at these meetings was approximately one hundred fifty, among whom were the following one hundred four members of the Society:

E. S. Allen, F. E. Allen, Baker, Beckwith, Bliss, Blumberg, Bradshaw, Brahana, Brooke, Bussey, C. C. Camp, Carmichael, Chapman, Chittenden, Coble, Crathorne, H. B. Curtis, Curtiss, Dalaker, H. T. Davis, Denton, Dickson, Doll, Dowling, Dresden, Escott, Everett, Eversull, Feldstein-Tartakovsky, Feltges, Fields, Fry, Gibbens, Glenn, Gouwens, Green, W. L. Hart, M. G. Haseman, E. R. Hedrick, Hildebrandt, Hoar, Hodge, Ingraham, Dunham Jackson, Kinney, Lane, Lennes, Logsdon, Lunn, McGaw, M. M. McKelvey, J. V. McKelvey, N. B. MacLean, MacMillan, March, Marshall, T. E. Mason, Meacham, B. I. Miller, E. B. Miller, G. A. Miller, Miser, C. N. Moore, E. H. Moore, E. J. Moulton, F. R. Moulton, A. L. Nelson, Newson, Olson, C. I. Palmer, Pitcher, J. F. Reilly, R. G. D. Richardson, Rider, H. L. Rietz, Risley, Roever, Roman, Schottenfels, Schweitzer, Shaw, W. G. Simon, Skinner, Slaught, Edwin R. Smith, Stecker, Steimley, Stouffer, E. L. Thompson, B. M. Turner, J. S. Turner,

Underhill, Wahlin, Walton, Warren Weaver, Wilczynski, Wilder, K. P. Williams, F. E. Wood, Roscoe Woods, J. W. Young, J. W. A. Young, Zehring, Ziwet.

The Council announced the election of the following persons to membership in the Society:

Professor Horace Seely Brown, Hamilton College;
Miss Bess Marie Eversull, University of Cincinnati;
Rev. Francis Joseph Gerst, Loyola College, Baltimore;
Professor Eleanora Harris, Central Missouri State Teachers College;
Professor Albert Anton Heinz, Tsing Hua College;
Miss Anna Margaret Mullikin, Oak Lane, Pa.;
Mr. Percy Charles Herbert Papps, Mutual Benefit Life Insurance Company, Newark;
Professor Harry Munson Showman, University of California, Southern Branch;
Mr. Willis Whited, Consulting Bridge Engineer, Pennsylvania State Highway Department;
Professor Emma Kirtland Whiton, University of Redlands.

Professor A. B. Coble was reelected a member of the Editorial Committee of the TRANSACTIONS, for a term of three years beginning October 1, 1922. The President was requested to appoint a committee to prepare nominations for officers and other members of the Council to be elected at the annual meeting in December.

On behalf of the committee that had collected a fund in honor of Professor E. H. Moore, Professor Arnold Dresden tendered the fund to the Society. The trust was accepted, under the conditions of the bequest. A complete account is given elsewhere in the present number of this BULLETIN.

The sessions of Friday morning and Saturday were devoted to the reading of the papers listed below. On Friday afternoon Professor A. B. Coble gave the symposium lecture on *Cremona transformations and applications to algebra, geometry, and modular functions*. The program for Saturday morning was made up of three papers delivered by invitation in special celebration of the anniversary of the Chicago Section.

On Friday evening a dinner was held at the Quadrangle Club at which one hundred six persons were present. Toasts were responded to by Professor Coble who spoke for

the Chicago Section, Professor Richardson who spoke for the Society, Professor Ziwet and Mrs. H. B. Newson who spoke for the early members of the Chicago Section, and Professor Hildebrandt who spoke on Professor E. H. Moore as mathematician, teacher and man. At the close of Professor Hildebrandt's remarks, Professor Dresden presented to Professor Moore a beautifully illuminated and bound manuscript containing a statement of the establishment of the Eliakim Hastings Moore fund, of which mention has already been made.

At the meetings of the Society, President G. A. Bliss presided, relieved by Vice-President R. D. Carmichael and Professors A. B. Coble, L. E. Dickson, and E. H. Moore.

Titles and abstracts of the papers read at the various sessions follow below. Miss Carlson's paper was read by Professor Jackson. The papers of Dr. Zeldin, Dr. Wiener, Professors Copeland, Glenn, Crum, and R. L. Moore, Dr. Woods, Dr. Speiser, Mr. Wilder, and Professor Hart were read by title. Professor Carr was introduced by Professor MacMillan, Dr. Speiser by Professor Dickson, and Mr. Wilder by Professor R. L. Moore.

1. Professor R. D. Carmichael: *Abstract definitions of the symmetric and alternating groups and certain other permutation groups.*

The most interesting of the theorems given in this paper is the following: If k operations are subject to the sole defining conditions that each is of order three and the product of each pair of them is of order two, then they generate a group which is simply isomorphic with the alternating group of degree $k + 2$. About a score of theorems are given, each of which affords the abstract definition of some infinite class of permutation groups, several of these theorems being devoted to various abstract definitions of the symmetric and alternating groups.

2. Professor D. R. Curtiss: *On the zeros of successive polars of a binary form.*

A theorem due to J. H. Grace states that if a binary form written as $f(z)$ in non-homogeneous notation is apolar to a

form $\phi(z)$, then it has a zero within any circle enclosing all the roots of $\phi(z)$. In the present paper it is shown that this is equivalent to the following statement, which has been proved in another way by Dr. J. L. Walsh: Let $z_1, z_2, \cdots, z_n$ lie within a circular region C and satisfy an equation

$$F(z_1, z_2, \cdots, z_n) = 0,$$

where F is symmetric in the z's and is a linear function of each z_i; then the equation $F(z, z, \cdots, z) = 0$ has at least one root in C. A proof is given by mathematical induction, and the theorem is shown to be a special case of one regarding the zeros of successive polars of a form which can be deduced from results due to Laguerre.

3. Professor D. R. Curtiss: *Relations between kindred P and Q functions.*

In his lectures on the hypergeometric function, Klein stated without proof the theorem that every Q function of order k could be expressed linearly with constant coefficients in terms of $k + 1$ kindred P functions. This statement would seem to have been based on a count of constants, and is, in fact, untrue in certain special cases. The present author gives a proof of its correctness, with a method for obtaining such representations, in case the families concerned are irreducible. This method also applies to many reducible cases. As a corollary it follows that in these reducible cases, as well as in the irreducible case, every $k + 2$ Q functions of order k which have the same exponents are linearly dependent. A similar examination is made of the relations between a Q function and a kindred P function and its derivative.

4. Professor C. N. Moore: *On the equivalence of the Cesàro and Hölder means for multiple limits.*

In a paper previously presented to the Society (this BULLETIN, vol. 25 (1919), p. 257) the author has established a theorem with regard to generalized limits in General Analysis which includes as special cases the Knopp-Schnee-Ford theorem with regard to the equivalence of the Cesàro and Hölder means for series, the analogous theorem of Landau for integrals, and other similar theorems dealing with simple limits. In the present paper this theorem is extended to the case of multiple limits, so that it now includes as special cases corresponding theorems relating to multiple series, multiple integrals, and other multiple limits. The main

part of the proof consists in the establishment of an identical relation between Cesàro and Hölder means for multiple limits, which, though considerably more complicated for the general case, is analogous to Schur's identity (MATHEMATISCHE ANNALEN, vol. 74 (1913), p. 452) for the case of simple series.

5. Miss Bess M. Eversull: *On convergence factors in triple series and the triple Fourier series.*

In this paper it is shown that if convergence factors satisfying certain conditions are introduced into a triple series summable ($C1$), the resulting series will converge and will approach the value to which the original series is summable, as the convergence factors approach unity. It is also shown that the Fourier development of a function of three variables that is finite and integrable (Lebesgue) will be summable ($C1$) to the value of the function at every interior point of a region throughout which the function is continuous. The foregoing results are used to show that the formal developments that arise in discussing certain problems in the flow of heat really furnish the desired solution of the physical problem.

6. Professor E. B. Stouffer: *Independent sets of coaxial minors of determinants.*

It is known that there cannot be more than $n^2 - n + 1$ independent coaxial minors of a determinant of the nth order. The author determines two such sets of coaxial minors, and obtains the expressions for the elements of the determinant in terms of the minors belonging to each independent set. If one or more of the columns of the original determinant are replaced by corresponding columns from other determinants of the nth order with independent elements, a set of determinants is obtained for which a complete system of independent coaxial minors is determined. The independence of certain sets of sums of coaxial minors of determinants is also proved. This is of importance in invariant theory.

7. Professor P. R. Rider: *On the minimizing of a class of definite integrals.*

Several problems in the calculus of variations lead to the consideration of a definite integral of the form $\int_{x_0}^{x_1} [(1+y'^2)^m/y'']dx$. These are of peculiar interest because the second derivative appears in the integrand, and comparatively few problems of that kind have been completely solved. For example,

Euler's historic problem of finding the curve which with its evolute encloses a minimum area gives rise to the particular case $m = 2$ in the above integral. The case $m = 1$ arises in the problem, treated by Rider and Dunkel, of obtaining the curve which with its caustic encloses a minimum area. In finding the curves of minimum mean radius of curvature with respect to the arc and with respect to the abscissa, we are led to the cases $m = 2$ and $m = 3/2$ respectively. In this paper the author obtains the equations of the extremals or minimizing curves for these cases, and proves sufficient conditions for a minimum. The question of the determination of the arbitrary constants that occur in the equations of the extremals is taken up, and it is shown that an extremal can always be passed through two given points and be made to have prescribed slopes at these points. Various cases of variable end conditions are considered. The theory is illustrated by means of the curve of minimum mean radius of curvature with respect to x.

8. Miss Elizabeth Carlson: *On the approximate representation of periodic functions of two variables.*

It has been proved by D. Jackson (TRANSACTIONS OF THIS SOCIETY, vol. 22) that if $f(x)$ is a given function of period 2π, if n is a given positive integer, and if m is a real number greater than 1, then there exists one and only one trigonometric sum $T_{mn}(x)$ of order n, for which the integral of the mth power of the absolute value of the difference between $f(x)$ and $T_{mn}(x)$ reaches a minimum. The corresponding theorems for functions of two variables are proved in this paper. Also, a sufficient condition for the convergence of $T_{mn}(x,y)$ to $f(x,y)$ is found to be that $\lim_{\delta=0} \omega(\delta)/\delta^{2/m} = 0$, where $\omega(\delta)$ is the modulus of continuity of $f(x,y)$.

9. Professor G. A. Miller: *Substitution groups whose cycles of the same order contain a given number of letters.*

The total number of letters in all the cycles of order h contained in the group G may be found as follows: Select a set of cycles of order k composed of all the different cyles of this order found in G. Let λ be the number of the complete sets of conjugates under G contained in this set. The total number of letters in all the cycles of order k found in G is then λg, g being the order of G. The triply transitive group of degree 6 and order 120 is the only group which is not

symmetric but has the property that the total number of letters in all of its cycles of the same order is exactly equal to the order of the group for every cycle which appears in the group. A necessary and sufficient condition that a transitive group of degree n contain a complete set of distinct conjugate cycles whose total number of letters is less than the order of the group is that the class of this transitive group be less than $n - 1$. There are exactly six transitive groups which are not alternating but have the property that the total number of letters in all the cycles of the same order save one is the order of the group, while for this one the total number of letters is twice the order of the group.

10. Dr. S. D. Zeldin: *Conformal transformations of linear homogeneous difference equations and their invariants.*

In this paper Dr. Zeldin shows that the most general conformal transformation which leaves invariant a linear homogeneous difference equation of the type $p_0(v)u_{v+n} + p_1(v)u_{v+n-1} + \cdots + p_n(v)u_v = 0$, where u and v are the circular coordinates of a point in two dimensional space, is of the form $v = \varphi(\xi)$, $u_v = cu_\xi$, where $\varphi(\xi)$ satisfies one of the two equations $\varphi(\xi + 1) - \varphi(\xi) = \pm 1$, and c is an arbitrary constant. He also determines the functions of the p's which form independent absolute invariants.

11. Dr. Norbert Wiener: *A new form of integral expansion.*

The author develops the formula

$$f(x) = \lim_{n=\infty} \frac{1}{n!\int_{-1}^{1} e^{1/(x^2-1)}dx} \int_0^1 \frac{d^{n+1}}{du^{n+1}} \{[(u-x)^n + (u+x)^n]\int_{-1}^{u} e^{1/(u^2-1)}du\} f(u)du$$

for even functions $f(x)$ which are uniformly analytic over $(-1, 1)$. He discusses the conditions of validity of this formula, contrasting it with Fourier expansions. He shows that if it is valid for $x = 0$, $f(x)$ is uniquely determined by its values in the neighborhood of $x = 1$.

12. Professor Lennie P. Copeland: *Note on certain semi-invariants of n-lines.*

In this paper the semi-invariantive conditions that one n-line be a pole-polygon of a second n-line are determined. It is proved that there exist $3 + (m - 2)(m - 1)$ necessary and sufficient conditions that two n-lines be mutual pole-polygons. If one n-line is a pencil, the second is also, and they have coincident vertices.

13. Professor O. E. Glenn: *Residues of figurate numbers.*

Systems of residues (modulo p, a prime) may be developed as follows: Select a sequence (r) of integers, infinite one way, satisfying any law, and employ its terms as the initial numbers of appropriate orders of differences in a table of differences. The numbers in this table are of the form

$$N = \sum_j r_j \binom{m}{j}$$

and their residues are arranged, in general, in symmetrically related geometric figures that may be delimited by formulas. Thus we obtain, for instance, all numbers of the type of N which are divisible by p. Among the results of the paper is given a determination of all combinatory numbers

$$\binom{m}{j}$$

which are divisible by p^w and not by p^{w+1}.

14. Professor W. L. Crum: *Inter-variate correlation and the successive measures of dispersion in an ordered statistical series.*

In this paper formulas are developed for calculating the coefficients of correlation between the variates and (1) those next preceding, (2) those preceding by two time-intervals, and so on up to k time-intervals. It is shown that these formulas are expressible in terms of the standard deviations of the series of original items and of their successive finite differences. Several of the correlation coefficients are worked out for particular illustrations from historical statistics. The writer then presents the hypothesis that we may measure dispersion in an ordered series by a succession of numbers, beginning with the standard deviation and including the successive inter-variate correlation coefficients as far as we like. If the correlation coefficients fall off rapidly, the series is highly unstable; if they remain nearly equal to the basic coefficient, the series is relatively stable.

15. Professor W. L. Crum: *Inter-variate partial regression equations in an ordered statistical series.*

This paper seeks to examine the phenomena of fluctuation in an ordered series from a somewhat different point of view from the usual attempt to discover fundamental periodicities. Instead of considering that there is a more or less closely

periodic tendency for values of the variable to repeat themselves, the paper studies the hypothesis that there is an average tendency for the items of a series to bear a definite relation to the items next preceding, to those next-but-one preceding, etc. The obvious method for discovering such a relation is by the use of partial regression equations, and general properties of these are developed for the kind of series under consideration. Application of the theoretical results is made to certain historical economic series with a view to testing the validity of the hypothesis and to determining its practical utility in the problem of prediction in time series.

16. Professor R. L. Moore: *Concerning relatively uniform convergence.*

In this paper the author shows that if a sequence of measurable functions converges at every point of an interval I, then it converges relatively uniformly over some subset of I whose measure is identical with that of I.

17. Professor R. L. Moore: *On the cut-points of continuous curves and of other connected point sets in space of two dimensions.*

A proper subset K of a point set M is said to disconnect M in the strong sense provided $M - K$ is not connected. It is said to disconnect M in the weak sense provided not every two points of $M - K$ can be joined by a *closed* and connected subset of $M - K$. The present paper contains, among other results, the following:

(I) No bounded, closed and connected point set M contains a closed and connected subset K which contains an uncountable set of points each of which disconnects M, but not K, in the strong sense.

(II) In order that a closed, connected and bounded point set M shall be a continuous curve which contains no simple closed curve it is necessary and sufficient that every closed and connected subset of M contain uncountably many points each of which disconnects M (in the strong sense).

(III) In order that the continuous curve M shall contain no simple closed curve it is necessary and sufficient that if K denotes the set of all those points $[X]$ of M such that X disconnects M then no subset of K shall disconnect M, even in the weak sense.

18. Professor F. E. Carr: *A solution of a spinning oblate spheroid two body problem.*

The problem deals with the precessional movement of the spheroid and was suggested by the non-rigorous treatment which the earth's precession received from Poisson, Serret and others. The solution for p, the precession, has the form $p = P_0 t + P$, where P_0 is a power series with constant coefficients and P a power series with periodic coefficients, in a parameter denoting the eccentricity of the spheroid. The solution for the motion of the second body is analogous to and a generalization of one of the solutions obtained by W. D. MacMillan, and reduces to that solution for a special case. The solution is not general, but arises from a special set of initial conditions. When applied to the system of the earth and moon and the earth and sun, the solution gives results that agree closely with observed values.

19. Dr. Roscoe Woods: *The elliptic modular functions associated with the elliptic norm curve E^7.*

Professor Bessie I. Miller has discussed (TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), p. 259) the elliptic norm curves E^n for $n = 3$, 4, and 5, for which cases the genus of the associated modular group is zero. In the present paper, which will appear in full in an early number of the TRANSACTIONS OF THIS SOCIETY, the case $n = 7$, for which the genus is 3 and which is fairly typical of the general case, is considered.

20. Dr. Andreas Speiser: *Die Zerlegung von Primzahlen in algebraischen Zahlkörpern.*

In this paper, which will appear in an early number of the TRANSACTIONS OF THIS SOCIETY, the problem of the decomposition of a prime number in an algebraic number domain is shown to be identical with a certain problem in the theory of linear substitutions in a Galois field.

21. Professor G. A. Bliss: *A boundary value problem in the calculus of variations.*

For problems of the calculus of variations with fixed end-points it has been shown by several authors that the Jacobi necessary condition for a minimum is closely associated with a boundary value problem of the linear differential equation theory. In the present paper an analogous situation is studied for a case when the end-points are variable, and the correspondences between the two theories are exhibited.

22. Professor J. F. Reilly: *Certain generalizations of osculatory interpolation.*

The Sprague-King formula for osculatory interpolation makes use of differences to order five, and requires that the curve have second order contact with the partial interpolation curves at the extremities of the interval to which the formula is applicable. The author shows how this formula can be generalized first to make use of differences to order $2h + 1$, where h is a positive integer; and second to require that the curve have contact of order k with the partial interpolation curves. Further, it is indicated how these generalizations apply to formulas employing central differences of four different types: (1) the ordinary central differences of y_h; (2) the central differences relative to the interval between y_h and y_{h+1}; (3) the odd central differences relative to the interval between y_h and y_{h+1}, and the even central differences of y_h, as used by Gauss and Karup; and (4) the even central differences of y_h and y_{h+1}, as used by Everett and Buchanan.

23. Professor Arnold Dresden: *A report on the scientific work of the Chicago Section*, 1897–1922.

This paper appears in the present number of this BULLETIN.

24. Professor J. B. Shaw: *On functional transformations.*

The first part of the paper is concerned with the structure of linear operators of the type representable by integral transformations, the second part with functions of these operators, and the third part with algebras of such operators built upon a given fundamental complex.

Examples are used to introduce the notions such as: (1) $\int_{-1}^{+1}(9x^2y^2 - 3x^2 - 3y^2 + 2xy + 2)()dy$ with fundamental functions 1, x, $(3x^2 - 1)$. (2) $\int_{-1}^{+1}(15x^2y^3 - 9x^2y + 3xy^2 - 5y^3 + 4y - x)()dy$ with fundamental functions 1, x, $(3x^2 - 1)$, $(5y^3 - 3y)$. In case (1) the transformation converts the functions into numerical multiples of themselves; in case (2) it converts each into a multiple of the preceding, and 1 into 0. (3) $\int_0^1[x(1 - y), y(1 - x)]()dy$ with normalized fundamental functions $\sqrt{2}\sin n\pi y/n^2\pi^2$, a denumerable infinity. (4) $\int_0^\infty \frac{1}{2}e^{-|x-y|}()dy$ with normalized fundamental functions $\sqrt{2}\cos(x\cot\alpha - \alpha)(\sqrt{\pi}\sin\alpha)$, a continuous infinity of fundamental functions. (5) $\int_0^x$ and $()dy$ with the

fundamental functions $x^{\alpha+n}$ where $0 \leqq \alpha < 1$, and n runs from 1 to ∞, integrally.

For each of these operators an identity transformation exists which gives the so called expansion of the function in terms of the fundamental functions.

In the second part of the paper the real character of the various solutions of integral equations is brought out, such as the Neumann solution, Fredholm's solution, Volterra's solution, and the linear algebra solution. In the third part the use of a given set of fundamental functions belonging to a given transformation in determining other related transformations is shown.

25. Professor E. H. Moore: *On the determinant of an hermitian matrix with quaternionic elements. Definition and elementary properties with applications.*

This paper will appear in an early number of this BULLETIN.

26. Dr. T. C. Fry: *Trigonometric expansions of aperiodic functions.*

At the summer meeting of the American Mathematical Society at Wellesley in September, 1921, a discussion arose regarding methods for determining the amplitudes of sinusoidal components in functions which are not themselves periodic. In the course of this discussion a paper by W. L. Hart (*On trigonometric series*, ANNALS OF MATHEMATICS (2), vol. 18, p. 99) was frequently mentioned and the criticism was made that the restrictions imposed by Hart rather seriously interfered with the usefulness of the results. In the present paper the author attacks this problem from the standpoint of the theory of divergent limits and removes the most serious of these restrictions.

27. Mr. R. S. Hoar: *Mathematical paradoxes involved in the new Bucyrus gasoline shovel.*

The new gasoline shovel just put out by the Bucyrus Company is the first one-engine shovel of any size. Its peculiar construction involves enough apparent mathematical paradoxes to furnish the material for a large part of a course in mechanics. For example, the digging part of the shovel is forced against the bank by the weight of the bank, rather than by the engine. Also, it has a one-part hoist which is

apparently a two-part hoist, and the direction of the hoisting force is at an angle of about 45 degrees from its apparent direction. The object of this paper is to place these paradoxes at the disposal of teachers of mechanics, and to inject a bit of mathematical recreation into the programme.

28. Professor E. W. Chittenden: *On permutable quadratic forms in infinitely many variables.*

The first part of the paper contains a determination of the properties of the most general limited quadratic form $K(x)$ in infinitely many variables permutable with a given limited form $H(x)$. The results obtained are applied in the second part of the paper to obtain the solutions of non-linear matricial equations.

29. Professor J. S. Turner: *A fundamental system of invariants of a modular group of transformations.*

In this paper the author considers the group H of all linear homogeneous transformations $x' \equiv ax + by$, $y' \equiv cx + dy$, $ad - bc \equiv 1 \pmod{p^2}$, where a, b, c, d are integers and p is an integral prime, and shows that any invariant $I(x, y)$ of this group is an invariant of the group G: $x' \equiv a_1x + b_1y$, $y' \equiv c_1x + d_1y$, $a_1d_1 - b_1c_1 \equiv 1 \pmod{p}$, where a_1, b_1, c_1, d_1 are integers. It is proved $I(x, y) \equiv R(L, Q) + pF(x, y) \pmod{p^2}$, where L, Q are the known fundamental invariants of the group G, and R, F denote rational and integral functions of their arguments with integral coefficients. By means of the transformation $x' \equiv x + py$, $y' \equiv y \pmod{p^2}$, it is then proved that $R(L, Q) \equiv R_1(L^p, Q^p)$, $F(x, y) \equiv F_1(L, Q) \pmod{p}$. Finally, it is proved that L^p, Q^p, $pL^\alpha Q^\beta$, where $\alpha, \beta = 0, 1, \cdots, p - 1$, α, β, not both zero, form a fundamental system of (independent) invariants of the group H.

30. Professor F. H. Hodge: *Note on a generalization of the strophoid.*

A circle tangent to the Y axis is drawn. This circle plays the same rôle in the determination of the curve in question as that taken by the Y axis in the ordinary construction of the strophoid. This leads to a curve of the sixth order. If the radius of the circle is increased indefinitely, this curve approaches the strophoid as the limiting case.

31. Professor E. P. Lane: *Ruled surfaces of Green-reciprocal congruences.*

The author considers an arbitrary non-developable surface S and associates with this surface an arbitrary Γ'-congruence and the reciprocal Γ-congruence, in the sense of G. M. Green. Corresponding to an arbitrary curve on S there is a ruled surface R' of Γ'; the lines of Γ reciprocal to the generators of R' form the corresponding ruled surface R. The present paper investigates the properties of such corresponding ruled surfaces. The formulas herein developed furnish, in particular, a very brief and elegant method of determining those ruled surfaces R' and R which are developable. The theorem of Green, that the conjugate of a Γ-tangent intersects the corresponding generator of the corresponding developable R in a focal point, is generalized in the form: *The conjugate of a tangent to the curve on S that corresponds to an arbitrary ruled surface R of the Γ-congruence intersects the corresponding generator of R at the point where R is touched by the corresponding tangent plane of S.* A class of curves called *intersector curves* is defined for R' and R. Applications are made to asymptotic ruled surfaces R' and R, which correspond to asymptotic curves on S. These, with the canonical quadric, serve to characterize the pair of covariant reciprocal congruences defined, in Green's notation, by the conditions $\alpha_u = \beta_v = 0$.

32. Professor K. P. Williams: *The Laplace-Poisson mixed equation.*

The author investigates the analytic character of the formal solutions which Borden has obtained for the Laplace-Poisson mixed equation

$$f'(x+1) + p(x)f'(x) + q(x)f(x+1) + r(x)f(x) = 0.$$

The asymptotic form of the solutions is also considered.

33. Mr. H. T. Davis: *A criterion from integral equations relating to the existence of solutions for the one-dimensional boundary value problem.*

In this paper the known existence of solutions of a linear homogeneous differential equation with a parameter of the Sturmian type for one set of boundary conditions is used to prove the existence of solutions which satisfy a second set of boundary conditions. If $G_1(x, t)$ is the Green's function for the first system, and $G_2(x, t)$ the Green's function for the

second system, then $\int_a^b [G_1(x, x) - G_2(x, x)]dx \neq 0$ is shown to be a sufficient condition that the values of the parameter for which solutions exist for the two problems shall either coincide or alternate with one another.

34. Mr. H. T. Davis: *A general criterion relating to the existence of solutions for the one-dimensional boundary value problem.*

Suppose F_1 is the determinant whose zeros are characteristic values of one boundary value problem, and F_2 the determinant belonging to a second boundary value problem. The product $F = F_1 \cdot F_2$ is used to furnish a criterion for the existence of solutions for one system, supposing the existence of solutions has already been proved for the other. An application is then made to the general self-adjoint system of second order.

35. Mr. R. L. Wilder: *A continuous curve in the rôle of a space.*

The author shows that if the set of points constituted by a continuous curve is regarded as a space S, then if certain terms are suitably defined, many theorems which are true in ordinary space hold true for the space S.

In particular, it is shown that Theorems 1–16 of R. L. Moore's paper *On the foundations of plane analysis situs* (TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), pp. 131–164) hold true. In addition, the following theorems are proved: (1) If R is a domain with respect to S, and B, the boundary of R with respect to S, is connected im kleinen, then any point P in B can be joined to any point in R by a simple continuous arc lying wholly in R except for the point P. (2) In order that a closed and bounded point set shall be connected im kleinen, it is necessary and sufficient that it be the sum of a finite (or vacuous) set of continuous curves, together with a finite (or vacuous) set of isolated points.

36. Professor N. J. Lennes: *Continuous transformations in analysis situs.*

For a given Jordan curve j and a fixed point P within it, this paper describes the construction of a set $[j]$ of Jordan curves such that (1) every curve lies within j and contains P as an interior point; (2) through every point within j, except P, there is one and only one curve of the set $[j]$. If any curve j_1 of the set $[j]$ is removed, the remainder of the set consists

of two sets of curves $[j]_1$ and $[j]_2$ such that (*a*) no point in either set is a limit point of points in the other; (*b*) every point of j_1 is a limit point of points in $[j]_1$ and also of points in $[j]_2$. The set of curves $[j]$ is said to constitute a *continuous transformation* of any one of its curves into any other or into the point P.

Such a set can be constructed transforming any plane curve into any other provided the curves have no point in common and one of the curves does not have a point interior to and another point exterior to the other curve.

The problem is proposed to construct a continuous transformation of a *doubly closed continuous curve not lying in a plane* into a plane Jordan curve. This problem, connected with the theory of "knots," is not completely solved.

37. Professor N. J. Lennes: *On the foundations of the theory of sets.*

In the MATHEMATISCHE ANNALEN, vol. 65, E. Zermelo gives a set of axioms for sets (Mengen), one of his purposes being to formulate a theory which should not involve the contradiction that has been shown to follow from Cantor's classical definition of "Menge." The purposes of the present paper are: (1) to show that though it follows from Zermelo's axioms that there are sets having cardinal numbers such as that of the continuum and higher, it cannot be shown that an arbitrary collection of objects more than finite in number is a set; (2) to modify Zermelo's theory so as to identify as a set any collection of objects having the same cardinal number as that of some Zermelo set; (3) to study the independence of Zermelo's axioms. It is proved that except for the "axiom of choice" (Axiom der Auswahl) Zermelo's axioms are independent. Similar independence proofs are given for the axioms with which it is proposed to replace Zermelo's set.

38. Professor L. E. Dickson: *An error in the theory of differential equations by Lie's method.*

In Lie-Scheffer's *Differentialgleichungen*, pp. 412–433, it is proved that every differential equation of the second order in two variables which is invariant under two infinitesimal transformations generating a two-parameter group can be integrated by quadratures, except in the last of four cases, when it is also necessary to integrate an auxiliary differential equation of the first order. But the last case is not excep-

tional, and also requires only quadratures. The oversight was due (p. 424) to not making full use of the hypothesis $(U_1U_2) \equiv U_1f$ of that case. In fact, the long Chapter 18 can be compressed into two pages by noting that under the former hypothesis or the remaining one $(U_1U_2) \equiv 0$, the infinitesimal transformation U_2f leaves invariant the partial differential equation $U_1f = 0$, whence a solution ϕ of the latter may be found by quadratures. When ϕ and y are taken as new variables, U_1f becomes $F\partial f/\partial y$, which is reduced to $\partial f/\partial y_1$ by choice of a function y_1 of ϕ and y found by an obvious quadrature. It is now a simple matter to obtain the four types of canonical forms of U_1, U_2 by quadratures only.

39. Professor L. E. Dickson: *Present status of the history of the theory of numbers.*

The purpose of this preliminary communication is to announce (1) that the third volume of the writer's *History of the Theory of Numbers*, entitled *Arithmetical theories of quadratic and higher forms*, is being put in type for the Carnegie Institution of Washington, and (2) that the publication will soon be undertaken by the National Research Council of a report prepared by Professors Dickson, H. H. Mitchell, Vandiver and Wahlin on algebraic numbers and related topics, chiefly from 1894 to date, with the primary aim to supplement the report by Hilbert in the yearbook of the German Mathematical Society for 1894.

40. Professor W. L. Hart: *The determination of a seasonal variation.*

Let $y = f(t)$ represent a statistical table, in which $t = 0, 1, \cdots, 12k$ months, where $t = 0$ is January, first year, $t = 15$ is March, second year, etc. The method of monthly means for determining the seasonal variation present in the function $f(t)$ consists of forming a January entry equal to the arithmetic mean of all the given January values, $\cdots$, a December entry equal to the mean of the December values. The author states certain useful properties of this method. He also gives an obvious justification of its use in the case of economic data, affected by the business cycle, by virtue of a certain well known property of the trigonometric functions. Attention is called to the relatively complicated character of the method for determining seasonal variation which has been used by Professor W. M. Persons, and an example is considered which illustrates the possible inaccuracy of his method.

41. Dr. V. D. Gokhale: *Concerning compact Kürschák fields.*

In the paper *Ueber Limesbildung und die allgemeine Körpertheorie* (JOURNAL FÜR MATHEMATIK, vol. 142), Kürschák sets up abstract fields with a modulus ("bewertete Körper"). This modulus plays, in the general field, essentially the same rôle as the absolute value in the fields of classical analysis, viz., real number system, complex number system, etc. Kürschák proves that every abstract field of this type has the (smallest) algebraically closed and perfect extension.

In the present paper the author sets up the notion *compactness.* This notion is analogous to M. Fréchet's compactness and to the J-compactness in E. H. Moore's General Analysis. It is a generalization of the following property in the point set theory: Every infinite set of points in a bounded domain has at least one condensation point. He then studies the properties of algebraically closed and compact fields and compactness under the adjunction of algebraic elements. Using Ostrowski's results, he proves the theorem that the smallest algebraically closed extension of a compact field is compact if and only if it can be obtained by adjoining a single algebraic element. The last part of the paper develops a complete existential theory of the four properties: (1) having a characteristic other than zero, (2) algebraic closure, (3) perfection and (4) compactness. Out of the $2^4 = 16$ possibilities 11 are shown to be existent and the remaining 5 nonexistent.

42. Professor W. H. Roever: *A second mechanism for illustrating lines of force.*

Several years ago the author designed a mechanism for illustrating certain systems of lines of force and stream lines (see ZEITSCHRIFT FÜR MATHEMATIK UND PHYSIK, vol. 62 (1914), and also BULLETIN OF THE MOUNT WEATHER OBSERVATORY, vol. 6 (1914), Part 5). In the mechanism exhibited at this meeting, one of the two wheels of the former mechanism is replaced by a striped curtain which moves with uniform linear velocity just behind a uniformly rotating spoked wheel. For sufficiently rapid motions of the curtain and wheel the paths of the points of intersection of the spokes of the wheel with the stripes of the curtain become distinctly visible. These curves were shown to be identical with lines of force of the field resulting from the introduction of a source (or a sink) in an originally constant field of force.

ARNOLD DRESDEN,
Secretary of the Chicago Section.

R. G. D. RICHARDSON,
Secretary of the Society.

A REPORT ON THE SCIENTIFIC WORK OF THE CHICAGO SECTION, 1897–1922 *

From December, 1896, forward, meetings were held in or near Chicago twice a year regularly, in the spring and during the Christmas holidays.

The number of papers presented at these meetings of the Section is 1,102. If we classify these papers on the basis of the encyclopedic classification, the result is as follows:

Arithmetic and Algebra	335	or 30 per cent.
Analysis	380	" 35 "
Geometry	248	" 22 "
Mechanics (incl. Math. Physics)	93	" 8 "
History, Pedagogy, Philosophy	46	" 4 "

It is found that the 335 papers on arithmetic and algebra were presented by 84 authors, giving an average of 4 papers per author; the 380 papers in analysis, by 135 authors, which is slightly less than 3 papers per author; the 248 papers in geometry, by 118 authors; in mechanics 93 papers, by 33 authors; in history, etc., 46 papers, by 18 authors. While thus the average number of papers presented by each author in each separate field is between 3 and 4, this average is very misleading. For in the first place, the same author occurs frequently in more than one group, the total number of authors being 278, giving a general average of 4 papers per author. But also this figure gives a very wrong impression of the relative distribution of the papers. For out of the 278 authors, about one half, exactly 133, presented one paper each, i.e., about 10 per cent of the papers. More striking still is the fact that there is a group of 10 authors, constituting less than 4 per cent of the total number, who have contributed 329 papers, which is about 30 per cent of the total; and finally, there is one author who has to his credit 70 of the titles, or over 6 per cent of the total. In view of such extremes, averages mean very little. If one groups the authors according to the number of papers they have presented, the

* Parts of a report presented before the Society on April 15, 1922.

number in each group decreases very rapidly as the number of papers per author increases. The result is as follows:

133	authors	having	presented	1	paper	each	.....	133
43	"	"	"	2	"	"	.....	86
24	"	"	"	3	"	"	.....	72
12	"	"	"	4	"	"	.....	48
11	"	"	"	5	"	"	.....	55
14	"	"	"	6	"	"	.....	84
5	"	"	"	7	"	"	.....	35
3	"	"	"	8	"	"	.....	24
9	"	"	"	9	"	"	.....	81
1	"	"	"	10	"	"	.....	10
3	"	"	"	11	"	"	.....	33
5	"	"	"	12	"	"	.....	60
1	"	"	"	13	"	"	.....	13
1	"	"	"	14	"	"	.....	14
1	"	"	"	15	"	"	.....	15
2	"	"	"	16	"	"	.....	32
1	"	"	"	19	"	"	.....	19
1	"	"	"	20	"	"	.....	20
1	"	"	"	21	"	"	.....	21
1	"	"	"	24	"	"	.....	24
1	"	"	"	27	"	"	.....	27
1	"	"	"	32	"	"	.....	32
1	"	"	"	35	"	"	.....	35
1	"	"	"	39	"	"	.....	39
1	"	"	"	42	"	"	.....	42
1	"	"	"	70	"	"	.....	70

The 10 authors coming last in this group are:

W. D. MacMillan	G. A. Bliss	F. R. Moulton
E. H. Moore	E. J. Wilczynski	R. D. Carmichael
A. R. Schweitzer	J. B. Shaw	G. A. Miller
L. E. Dickson		

If now we inquire as to the publication of these papers, we find that in arithmetic and algebra there have been published 213 out of 335 (64 per cent), in analysis 217 out of 380 (57 per cent), in geometry 147 out of 248 (59 per cent), in mechanics 49 out of 93 (53 per cent) and in history, etc., 23 out of 46 (50 per cent)—a total of 649 out of 1,102 (59 per cent).

I shall not attempt to evaluate the relative importance for the work of the Section of published versus unpublished papers. Doubtless there are many papers that have fulfilled their total possible usefulness to author and public when once presented. I do not believe therefore that 100 per cent publication is an end to be too diligently pursued. The

meetings of the Section would lose a good share of their value if there were no room for airing views, proposing questions, and making suggestions which might not be suitable to appear in print. On the other hand, a small ratio of published to presented papers would not be a sign of the most wholesome development. The figures presented above give but a rough idea of the general character of the published material because they include a variety of journals of varying grade and standard. For that reason it seemed worth while to single out for separate mention the group of papers published in the TRANSACTIONS. This group consists of 166 papers, which is about one fourth of the published papers, or 15 per cent of the total number of papers presented. It is interesting to note that not a single one of the 21 completed volumes of the TRANSACTIONS is without several Chicago Section papers, the number averaging 8 per volume and running as high as 12 and 13 in some volumes.

I have thus dwelt somewhat at length upon the papers offered for presentation at the meetings of the Chicago Section, because I feel that this aspect of our meetings is perhaps most widely useful. For a mathematician to know that he can find a group of more or less informed colleagues who will at least listen to what he has to say about his own work, and perhaps comment on it with a measure of understanding of what he is working for, encourage him if he deserves it, discourage him if it be otherwise, must be for him a stimulus of which each one of us can best appreciate the value from his own experience. This value, we will agree, I think, depends to some extent upon the quality and the quantity of our auditors. Since of the former of these characters it is difficult for me to speak, I shall make a brief statement about the latter. The attendance of our meetings is very variable. During the first ten years the number of members present never exceeded 30. Since then it has never fallen below 30, jumping up to 84 in December, 1908, when the A. A. A. S. met in Chicago, to fall back to 33 at the following meeting in April, 1909. Since then the attendance at the meetings has grown fairly

steadily to an average of about 50. A year ago the number went up to 83, and if our expectations are not disappointed, all previous records will be surpassed on the present occasion. (The attendance at the meeting of April 14 and 15 included 104 members.) But this growth in numbers is but an outward sign of an inner development. And this inner development was the result of careful and thoughtful nurture on the part of those who carried the responsibility.

At the meeting of April, 1897, the Chicago Section was formally organized with E. H. Moore as chairman, T. F. Holgate as secretary, and Professors Ziwet and Hathaway as members of the program committee. In December, 1905, Professor H. E. Slaught was elected secretary of the Section, and he continued to serve in that office for ten years, until December, 1915. No one can appreciate the devotion which he gave to his work as secretary more than his successor does, unless it be those who were associated with him on the program committee. It was during the second decade of its existence that the Section took its big strides forward and all those who, like myself, began to attend the meetings during that time know to how large an extent it was his work that led it forward.

The Section entered upon a period of vigorous development in 1905. A committee was appointed to consider means of improving the meetings of the Section. As a result of the work of this committee, there were held in December, 1907, joint meetings with Sections A and D of the A. A. A. S., including a symposium on the teaching of mathematics to engineers. The following year the Section appointed a committee to study the possibility of improving the character of the mathematical appointments in our colleges and universities, resulting in a report adopted by the Section and a tentative plan proposed by Professor Wilczynski and published in the BULLETIN. Joint meetings with the A. A. A. S. and with engineering organizations were held in Minneapolis in 1910, in Cleveland in 1912, in Columbus, Ohio, in 1915, in St. Louis in 1919, and in Toronto in 1921. In April, 1911,

the Society met in Chicago to hear Professor Bôcher's presidential address. In 1913 the Council of the Society took official notice of the growing importance of the meetings of the Chicago Section by resolving that "the meetings of the Chicago group are henceforth to be designated as meetings of the Society." From 1911 on the retiring chairman delivered an address. When voting in 1913 to make the chairman's address a permanent feature, the Section expressed the opinion "that there are too few papers giving a general survey of any field of mathematics." For a short period, time was set aside on each program for "informal notes and queries" in the hope that in that way more general participation in the discussion might be secured. These different efforts to give the meetings a broader scope led, in 1915, to a proposal by Professor Van Vleck that a symposium be arranged on some topic of wide interest. But it was not until 1917 that the proposal was carried into effect; at that time Professors Bliss and Hildebrandt gave lectures on the *Lebesgue Integral*. From that time on a symposium address has been given at the April meeting of each year.

ARNOLD DRESDEN,
Secretary of the Chicago Section.

ELIAKIM HASTINGS MOORE FUND

BY ARNOLD DRESDEN

A group of former students of Professor E. H. Moore, wishing to use the opportunity afforded by the twenty-fifth anniversary meeting of the Chicago Section to present to Professor Moore a testimonial of their respect and appreciation, brought together a fund to be used for furthering mathematical interest in this country. This fund was contributed to by one hundred and seventy-four persons, former graduate students of mathematics at the University of Chicago, or members of the American Mathematical Society definitely identified with its Chicago Section. On the evening of April 14, 1922, a beautifully bound and illuminated manuscript

containing a statement of the establishment of the Eliakim Hastings Moore Fund was presented to Professor Moore. The names of the contributors to the fund are preceded by the following statement:

"Conscious of the great influence which you have exercised upon the development of mathematical science throughout this country, particularly in the Middle West, during the last twenty-five years,

Admiring the outstanding qualities of your researches in various fields of mathematics,

Grateful for the inspiration and the encouragement which you have given to those who have come to the University of Chicago to study mathematics,

Recognizing the large contribution which you have made to the creation and the growth of the Chicago Section of the American Mathematical Society,

Deeply appreciative of the friendship which, during many years, you have shown toward those who have had the good fortune to know you,

The undersigned members of the American Mathematical Society, formerly students of mathematics at the University of Chicago, or members of long standing in the Chicago Section, have wished to use the opportunity afforded by the twenty-fifth anniversary meeting of the Chicago Section to present to you a testimonial, which is intended to link your name in the years to come with the development of mathematics in this country.

To this end they have contributed to a fund which is to be offered for trusteeship to the American Mathematical Society upon the following conditions:

1. The fund is to be known as the Eliakim Hastings Moore Fund.

2. The interest on the fund is to be used at the discretion of the Council of the Society, and upon the recommendation of a committee appointed from time to time for this purpose, in furtherance of such mathematical interests as

(*a*) The publication of important mathematical books and memoirs;

(*b*) The award of prizes for important contributions to mathematics;

it being further recommended that during the next ten years preference be given to the former, and that publication of Professor E. H. Moore's researches in General Analysis or other fields shall have precedence over all other claims.

3. The fund is to be kept intact by the American Mathematical Society, except in so far as it is used to aid in the publication of Professor Moore's researches. For this special purpose a part of the principal, not exceeding one third, may be used provided the interest on the remainder be allowed to accumulate until the fund has been restored to its original value."

At its meeting on April 15, the Council of the Society accepted the trusteeship of this fund as recorded elsewhere in the BULLETIN.

It is hoped that the Eliakim Hastings Moore Fund, which has now reached approximately $2,000, may be the nucleus of a much larger fund to be ultimately established and to be at the disposal of the American Mathematical Society for aid in the publication of important mathematical work. The fund will remain open for contributions at any time, and it is hoped that those who are interested and who had no opportunity to join in the testimonial at the time of its foundation may do so later. Contributions may be sent to the Secretary of the Society, Professor R. G. D. Richardson, Brown University, Providence, Rhode Island.

THE UNIVERSITY OF WISCONSIN.

NOTE ON THE DIVISION OF A PLANE BY A POINT SET*

BY E. W. CHITTENDEN

A plane set of points K is said to divide a plane S if the set $S - K$ is composed of two mutually exclusive domains S_1, S_2, of which K is a common boundary, where by domain is meant a connected open set. The condition that K be a simple closed curve or an open curve has been stated by J. R. Kline† in terms of the concept "connected im kleinen." In proving that the set K is a connected set, Kline employs the condition "connected im kleinen."

If we assume that K is bounded and that we have at our disposal the parallel and perpendicular straight lines of a number plane, then the connectedness of K is established by other writers, for example Hausdorff, *Grundzüge der Mengenlehre*, page 346, Theorem XII. Hausdorff calls attention, in a footnote to page 342, to the difficulty of extending his argument to the case of unbounded sets.

It seems in view of the importance of the theory of open curves as indicated by R. L. Moore‡ and of the importance in general of the fundamental theorems of plane analysis situs that it is of interest to show that the set K is connected, whether bounded or not, without the use of the restriction employed by Kline or of the properties of straight lines and rectangles. The present note is concerned, therefore, with the proof of the following theorem.

THEOREM. *Let K be a plane point set, S, the set of all points of the plane, and denote by S_1, S_2 two mutually exclusive domains such that*

$$S - K = S_1 + S_2.$$

Then if every point of K is a limit point of both S_1 and S_2, the set K is closed and connected.

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* Presented to the Society Nov. 26, 1921.

† *Concerning approachability of simple closed and open curves.* TRANSACTIONS OF THIS SOCIETY, vol. 21 (1920), pp. 451–458.

‡ R. L. Moore, *On the foundations of plane analysis situs.* TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), pp. 131–164. This paper will be referred to as "Foundations."

Proof. The set K is closed. For any limit point of K is evidently a common limit point of S_1 and S_2. But no limit point of S_1 can belong to the domain S_2, and likewise no limit point of S_2 can belong to S_1. Such a point must belong to K.

Assume that the set K is not connected. Then there exist two mutually exclusive closed subsets K_1 and K_2 of K such that $K = K_1 + K_2$. Let P_i denote a point of K_i $(i = 1, 2)$. We may enclose P_i in a region R_i which contains no point of K_{i+1}.* Let J_i be a simple closed curve lying in R_i and containing P_i as an interior point. Let P_{ij} be a point of S_j lying within J_i. Since S_j is a domain, there is an arc

$$P_{1j} X P_{2j}$$

lying entirely in S_j. Let A_{1j} be the last point which this arc has in common with J_1 and let A_{2j} be the first point which the arc has in common with J_2 after A_{1j} on $P_{1j} X P_{2j}$.

Since the point A_{ij} lies on the boundary of J_i, it may be connected with P_i by an arc $P_i X A_{ij}$ which, except for the end-point A_{ij}, lies entirely within J_i. Furthermore the arcs $P_i X A_{i1}$ and $P_i X A_{i2}$ may be constructed so that they have no common point besides P_i.

From the arcs so defined we construct a simple closed curve J,

$$J: P_1 A_{11} A_{21} P_2 A_{22} A_{12} P_1.$$

Let H_1 denote the set of all points of K_1 which are interior to J but not interior to J_1. The set H_1 is closed.

CASE I. No point of H_1 lies on J_1. Then the points of J_1 which lie in J lie in $S_1 + S_2$. There exist points of J_1 within J, since by Theorem 40 of the *Foundations* it is possible to join P_1 and P_2 by an arc $P_1 X P_2$ such that $\underline{P_1 X P_2}$† lies entirely within J. This arc must meet J_1 in at least one point. It follows readily that there is an arc $A_{11} X A_{12}$ of J_1 which lies in J and therefore in $S_1 + S_2$. This is contrary to Lemma A of the paper of J. R. Kline.‡

* The subscripts are to be reduced modulo 2.

† The symbol $\underline{A X B}$ denotes the set $A X B - A - B$, that is, the set of all points of the arc $A X B$ except its end-points.

‡ Loc. cit., p. 452. Every arc joining a point of S_1 to a point of S_2 contains a point of K.

CASE II. The curve J_1 contains a point of H_1. From the Heine-Borel property we may assume a finite set of regions

$$\overline{R}_1, \overline{R}_2, \cdots, \overline{R}_m, \tag{1}$$

covering H_1. We may without loss of generality assume that the boundary of each region $\overline{R}_k$ $(k = 1, 2, \cdots, m)$ is a simple closed curve $\overline{J}_k$. We will also assume that no point of J, J_2, or K_2 lies in or on the boundary of any of the regions $\overline{R}_k$. By hypothesis some of the regions $\overline{R}_k$ have points in common with the interior of J_1. Let

$$\overline{R}_1, \overline{R}_2, \cdots, \overline{R}_p, \quad (p \leqq m), \tag{2}$$

be a subset of the regions (1) such that $\overline{R}_1 + \cdots + \overline{R}_p$ forms, with the interior of J_1, a connected set. Then the curves $J_1, \overline{J}_1, \overline{J}_2, \cdots, \overline{J}_p$ form a finite family G of closed curves whose interiors form a connected set. By theorem 42 of the *Foundations* there is a simple closed curve $\overline{J}$ which satisfies these conditions: every point of $\overline{J}$ belongs to some curve of G: the interior of $\overline{J}$ contains the interiors of all the curves of the set G.

The curve $\overline{J}$ meets J in the points A_{1j} and in no other points. We may show as in Case I that there is a point X on $\overline{J}$ which is interior to J, and that the arc $A_{11}XA_{12}$ of the curve $\overline{J}$ lies within J.

We will show that no point of $A_{11}XA_{12}$ is in K. By construction no point of K_2 lies on any curve of the set G. Suppose Q, a point of K_1, lies on the arc $A_{11}XA_{12}$. Then Q must lie on one of the curves of the set G. If Q is on J_1, it must belong to H_1 since Q is interior to J. Consequently Q is in H_1 in any case and must lie in some region $\overline{R}_k$ $(k \leqq m)$ of the regions (1). Suppose that Q lies on $\overline{J}_q$ $(q \leqq p)$. Then R_k must contain an interior point of $\overline{J}_q$. But the interior of $\overline{J}_q$ is connected with R_1. Consequently R_k is connected with R_1 and is interior to $\overline{J}$. It follows that Q is an interior point of $\overline{J}$, contrary to hypothesis.

Since no point of K lies on $A_{11}XA_{12}$, we have obtained an arc connecting a point of S_1 with a point of S_2 which contains no point of K. This again contradicts Lemma A.

The proof that the set K is connected is completed.

THE UNIVERSITY OF IOWA

NOTE ON STEADY FLUID MOTION

BY S. D. ZELDIN

In a previous paper* I have shown how to find special invariant configurations of the projective and linearoid groups investigated by Wilczynski† in connection with steady fluid motion. It is the purpose of this note to show how the group whose general infinitesimal transformation is

$$Kf = u(x)\frac{\partial f}{\partial x} + v(x, y)\frac{\partial f}{\partial y} + w(x, y, z)\frac{\partial f}{\partial z}\ddagger$$

should be simplified in order to represent the steady motion of a fluid under the influence of forces possessing a potential.

If the external forces have a potential, then, as is known, the functions u, v, w must be such that the expression

$$Kudx + Kvdy + Kwdz$$

is a complete differential, or, what amounts to the same thing,

$$(1)\qquad \begin{aligned} \frac{\partial Ku}{\partial z} - \frac{\partial Kw}{\partial y} &= 0, \\ \frac{\partial Kw}{\partial x} - \frac{\partial Ku}{\partial z} &= 0, \\ \frac{\partial Ku}{\partial y} - \frac{\partial Kv}{\partial x} &= 0. \end{aligned}$$

Performing the operations indicated by equations (1) we get:

$$(a)\qquad \frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x} + u\frac{\partial^2 v}{\partial x^2} = 0,$$

$$(b)\qquad \frac{\partial u}{\partial x}\cdot\frac{\partial w}{\partial x} + u\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial x\partial y} + w\frac{\partial^2 w}{\partial x\partial z} + \frac{\partial v}{\partial x}\cdot\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial z} = 0,$$

$$(c)\qquad u\frac{\partial^2 w}{\partial x\partial y} + v\frac{\partial^2 w}{\partial y^2} + w\frac{\partial^2 w}{\partial y\partial z} + \frac{\partial v}{\partial y}\cdot\frac{\partial w}{\partial y} + \frac{\partial w}{\partial y}\cdot\frac{\partial w}{\partial z} = 0,$$

* JOURNAL OF MATHEMATICS AND PHYSICS (Mass. Inst. of Tech.), vol. 1 (1921), p. 54.

† TRANSACTIONS OF THIS SOCIETY, vol. 1 (1900), pp. 339–352.

‡ This class of groups has been investigated by Sophus Lie in connection with two-point invariants. See Lie-Engel, *Theorie der Transformationsgruppen*, vol. 3, Abtheilung 5.

which can be written in the form

$$(a) \qquad \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial x}\left(v\frac{\partial v}{\partial y}\right)=0,$$

$$(b) \qquad \frac{\partial}{\partial x}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=0,$$

$$(c) \qquad \frac{\partial}{\partial y}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=0.$$

From (a) follows at once that *Kv can be a function of y only.* From (b) we see that Kw must be a function of y and z alone, and, since by virtue of (c) Kw must be a function of x and z alone, it follows that *Kw can be a function of z only.*

If the fluid is incompressible, we have the equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0,$$

and therefore w is a *linear function* of z.

In the case of irrotational motion, since

$$\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=0, \qquad \frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=0, \qquad \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0,$$

we must have, in the above infinitesimal transformation Kf, *u a function of x alone, v a function of y alone,* and *w a function of z alone.* There exists then a velocity potential, say F, where

$$u=\frac{\partial F}{\partial x}, \qquad v=\frac{\partial F}{\partial y}, \qquad w=\frac{\partial F}{\partial z},$$

and the orthogonal trajectories of the family of surfaces

$$F=\int u(x)dx+\int v(y)dy+\int w(z)dz=\text{constant}$$

represent the stream lines. These stream lines are the intersections of the two families of cylinders obtained by solving the equations

$$\frac{dx}{u(x)}=\frac{dy}{v(y)}=\frac{dz}{w(z)}.$$

The separately invariant points will be found by solving the equations

$$u(x)=0, \qquad v(y)=0, \qquad w(z)=0.$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

TWO BOOKS ON ANALYSIS

Lezioni di Analisi Infinitesimale. By Giulio Vivanti. 2d edition. Torino, S. Lattes and Co., 1920. vii + 693 pages.

Lezioni di Calcolo Infinitesimale. By Ernesto Pascal. Milano, Ulrico Hoepli. Part I, 4th edition, 1919, viii + 330 pages. Part II, 4th edition, 1918, xii + 313 pages. Part III, 2d edition, 1918, xi + 325 pages.

Permitting oneself a variation on a well known theme, one could perhaps say to a nation: "Show me your fundamental course in analysis and I will tell you who you are mathematically." For doubtless, a valuable estimate of a nation's state of development in mathematics can be obtained by considering the form in which the fundamentals of the infinitesimal calculus are presented to the students of that nation.

If the two works under review may be taken as representative Italian courses, one must form a high estimate of the esteem in which mathematics is held in that country. In scope, point of view and method of approach, they are broad and scholarly. They cover practically the same ground except for Pascal's third volume which is devoted entirely to the calculus of variations and to the calculus of finite differences. They are not encyclopedic like the French cours d'analyse; neither are they written on the superficial plan of so many of our American college texts. From them a student can learn enough to be well prepared for special study in analysis, as well as for work in the applied sciences—he will at least have acquired that which he is to apply. Both intended for technical students, the authors do not hesitate to include the elements of the theory of point sets and other topics usually regarded in our classes as material unfit for the training of "practical men." In his preface, written May 1917, Pascal says "It is certain that through the profound changes which the critical spirit has made in the foundations of the calculus, even a course intended for those for whom mathematics is a means rather than an aim, cannot but use the new results which have been reached . . . it would therefore exhibit a shortsighted view and little esteem for the ability of the future engineer, to believe that it would be sufficient for them, at least if they can, to learn to operate the calculus in about the way in which a workman knows how to operate a machine made by others, and of which he does not know the inner connections."

Is not this a point of view worth the consideration of our teachers of engineering students? Not merely for the mathematical specialist, but for the person concerned with the applied sciences, a fundamental theoretical course, not merely a working course, is requisite if these applications are to be more than mere mechanical adaptations of the thoughts of others.

The two books differ in their method of treatment, inasmuch as Vivanti is in favor of and Pascal opposed to the fusion of differential and integral calculus. The *Lezioni* of Vivanti consists of the following six parts: I. Analytical introduction (90 pp.), II. Derivatives and integrals of functions of one variable (150 pp.), III. Derivatives and integrals of func-

tions of several variables (84 pp.), IV. Geometrical applications (183 pp.), V. Differential equations (138 pp.), VI. Calculus of variations (20 pp.).

In the analytical introduction are assembled those notions from the theory of functions of a real variable which form the irreducible minimum for a sound development of the calculus: real numbers, sequences and series, greatest lower and least upper bounds, continuity, uniform convergence, etc. This part reveals what an excellent expositor the author is. The reviewer noticed with pleasure several neat proofs, such as the one on page 39, in which it is shown that $\lim_{x \to c} f(x) = a$ implies that $\lim_{x \to c} b^{f(x)} = b^a$. In the arrangement of the material, in the working out of details, in the style and the form, a masterly hand shows itself. This section provides for the reader an enjoyment which I would only spoil by dwelling on the details.

With such preparation a thorough treatment of the calculus becomes possible. A curious slip occurs on page 140, where from the uniform convergence of a series of functions $f_i(x)$ on an interval (ab), the conclusion appears to be drawn that a series of functional values $f_i(x_i)$ must converge for arbitrary choice of the x_i. It is surprising to find in this book the notation $\partial f(\alpha, \beta)/\partial\alpha$ where $[\partial f(x, y)/\partial x]_{\alpha, \beta}$ is meant, a notation no longer used in our better texts and, in my judgment, very undesirable. Space is lacking for detailed mention of the many instances of elegance in treatment found throughout these parts of the book.

It seems curious that with such free fusion of the two divisions of the calculus, there should be such a sharp separation between the calculus and its geometric applications, to which the largest single part of the book is devoted. In the first 20 pages of this section we find an exposition of the elements of vector analysis which follows the methods of Burali-Forti and Marcolongo. Throughout the rest of the section these methods, as well as the ordinary methods, are used in the applications to differential geometry, which in many instances are improved and simplified in this way. The incorporation of the vector treatments, assembled in an appendix in the first edition of the *Lezioni*, as an organic part of the book in this new edition, bears witness to the vogue which the work of Burali-Forti and Marcolongo has given these methods.

In Part V, particular mention should be made of the simple and straightforward discussion of the singular integral and its geometrical interpretation, as well as of the treatment of the linear homogeneous equation with constant coefficients by the use of simple relations holding between the properties of the linear differential operator and the associated algebraic functions. A brief section devoted to the calculus of variations treats a few of the classical problems and merely mentions some of the others. Numerous examples are worked out in the text. For further exercises the reader should consult the companion volume, the author's *Esercizi*.* The misprints are very few and do not seriously mar the typographical excellence of this book.

* See the review of the book by R. C. Archibald in this BULLETIN, vol. 20 (1914), p. 482.

In the three volumes of Pascal's *Lezioni* on the other hand, misprints are quite numerous, particularly in volume 3, where some fifty were noticed on slightly over three hundred duodecimo pages. Enough has been said above in comparing the two works to make further discussion of the first two volumes of Pascal's *Lezioni* superfluous. The third volume is little different from the first edition of 1897 of which the first part appeared in a German translation by Schepp in 1899.* It appears to be a reprint of this earlier volume rather than a new edition. It is this which accounts for the apparent disregard of the important developments in the calculus of variations during the last twenty years. It still frankly represents the old school, so that a detailed criticism from the modern point of view, for which the book furnishes ample opportunity, would clearly be out of place. The extensive bibliographies inserted at various points form a valuable feature. It is surprising, however, that in the list of treatises on page 19, apparently revised since the earlier edition, Hadamard's *Leçons* is not mentioned, while there is mention made of a *Lehrbuch der Variationsrechnung* by Carathéodory and Zermelo, which, although announced repeatedly, has not yet appeared and does not occur on Teubner's later lists of future publications.

ARNOLD DRESDEN

SHORTER NOTICES

Allgemeine Theorie der Raumkurven und Flächen. By V. Kommerell and K. Kommerell. Vols. I and II, third edition. (Sammlung Schubert, XXIX and XLIV.) Berlin and Leipzig, Vereinigung wissenschaftlicher Verleger, 1921. 184 + 196 pp. 28 + 13 figs.

The third edition of these two volumes is so nearly a reprint of the second edition, which has already been reviewed in this BULLETIN,† that an account of its contents is quite unnecessary. The arrangement of material is precisely the same in the two editions, but many of the discussions have been slightly amplified in the later one and the few errors in printing have been corrected. The only new material appears in the derivation of curves from given properties and in the definition of the Christoffel symbols.

These two volumes should be intelligible to the student who has little training beyond the calculus, yet they present an excellent treatment of the essentials of differential geometry. The student who has read them should have no difficulty with the more extensive treatments of Bianchi, Eisenhart, Forsyth and others. Such a presentation of differential geometry as this by Kommerell and Kommerell, if available in English, would increase the teaching of that important subject to the advanced students in our American universities.

E. B. STOUFFER

* See the review by E. R. Hedrick in this BULLETIN, vol. 12 (1906), p. 172.

† Vol. 21 (1915), pp. 99–100.

Three Lectures on Fermat's Last Theorem. By L. J. Mordell. Cambridge, University Press, 1921. 30 pp.

The pamphlet consists of three lectures on Fermat's last theorem, given at Birkbeck College, London, in March 1920. The lectures, as stated in the preface, were intended primarily for persons with a mathematical training, but not necessarily for those who had made a special study of the theory of numbers. The work is divided into three chapters.

The first chapter contains a brief discussion of Fermat's work and the history of the theorem. It also contains the consideration of the early proofs of the impossibility of Fermat's equation $x^n + y^n = z^n$ for the cases $n = 3, 4, 5,$ and 7.

The second chapter is an exposition of Kummer's work in attempting to prove the theorem. Algebraic numbers and the arithmetic in an algebraic domain are discussed, showing how the failure of the unique factorization in such a domain necessitates the introduction of ideal numbers. The nature of these ideal numbers is very clearly and briefly presented. The author explains in an elegant manner the main points in Kummer's work leading to the proof of the impossibility of the solution of $x^p + y^p = z^p$ in the domain defined by the primitive pth roots of unity, when the class number of the domain is prime to p. The chapter ends by a mention of the more recent results based on Kummer's work.

In the last chapter the author discusses the methods of Libri and Sophie Germain and the results obtained by these methods, of which may be mentioned Dickson's proof of the impossibility of a solution of Fermat's equation in integers prime to p (p the exponent) when $p < 7000$.

G. E. WAHLIN

The Absolute Relations of Time and Space. By Alfred A. Robb. Cambridge, University Press, 1921. 80 pp.

This is really a contribution to the geometry of point events in which stress is laid upon the importance of the ideas of before and after. To illustrate his ideas, the author makes frequent use of cones, somewhat after the manner of Minkowski, and introduces the idea of conical order. The reviewer believes that a reader will find it helpful in studying Robb's logical analysis if he also uses the geometric representation of a point event by a closed surface completely surrounding the point. An event A may then be regarded as after an event B if the representative surface of A completely surrounds the representative surface of B. If the two representative surfaces intersect or are external to one another, the event A is neither before nor after B. The geometry thus visualized may be regarded as a wave-geometry, the representative surface of an event A being the locus, at the very beginning of things, of point events whose combined influence results directly in the occurrence of the event A. The simplest assumption one can make is that the representative surface is a sphere with A as center, but a more general assumption is suitable for Einsteinian geometry and for the geometry of light waves in a material medium or of sound waves in a windy atmosphere.

H. BATEMAN

Philosophy and the New Physics. An Essay on the Relativity Theory and the Theory of Quanta. By L. Rougier. Authorized translation from the author's corrected text of *La matérialisation de l'énergie* by M. Masius. Philadelphia, Blakiston, 1921. 16mo. 16 + 159 pp.

The title of this translation is somewhat misleading. That of the original, *La Matérialisation de l'Energie*, gives a better idea of the scope of this work, which is an excellent presentation of the way in which the old dualistic view of matter and energy has given way to the modern merging of these two concepts into each other. The foundation for the modern view was laid when Sir J. J. Thomson showed, in 1881, that the inertia of an electrically charged sphere is increased by its motion. This led to the conception of electromagnetic momentum in the ether, and the author shows how the theory of relativity has solved the difficulties which this conception involves when it was combined with the stationary ether required for optical phenomena.

The book was apparently written before the author was familiar with the general theory of relativity, for Einstein's first value for the deflection of the light from a star by the sun's gravitational field is given. This is, however, corrected in a note later on. The author appears to be satisfied that the displacement of the solar spectral lines towards the red, demanded by the theory of relativity, has been experimentally confirmed, although this is far from being true.

Coming to the theory of quanta, it is difficult, as the author shows, to avoid the conclusion, if the consequences of the theory of relativity be accepted, that radiation is propagated by means of discrete elements or quanta. But until it is shown that the differential equations of optics, which are wholly successful in interpreting all ordinary optical phenomena, are consistent, perhaps only to a first approximation, with a quantum theory of radiation, one can hardly agree with the author in his extreme view.

There is a wrong reference to Poincaré's paper on the dynamics of the electron on page 73. The correct one is given in the bibliography on page 154.

E. P. ADAMS.

Das Exzentrizitätsprinzip als Korrelat zur Relativitätstheorie. By K. M. Kohler. Vienna, Franz Deuticke, 1921.

In this seventy-page pamphlet the author presents the restricted theory of relativity in a novel form by studying the geometry of the one-parameter family of spheres

$$(x - vt)^2 + y^2 + z^2 = c^2t^2, \qquad t > 0.$$

His conclusions are in accord with Einstein's theory and his method is of interest. He spoils an otherwise interesting paper, however, by claiming to have found a means of distinguishing between absolute motion and rest by correlating these two states with the eccentricity and concentricity of the electro-magnetic waves emitted by the body in question. Since he shows clearly that to an observer moving with a moving body the waves emitted by it would appear to be concentric, his claim is not to be treated seriously. This claim is not, however, organically related to the remainder of the paper.

C. N. REYNOLDS, JR.

NOTES

At the meeting of the International Astronomical Union, held at Rome, May 2–10, 1922, Dr. W. W. Campbell, of the Lick Observatory, was elected president for the coming three years. Professor Tuliio Levi Civita was appointed chairman of the committee on relativity.

The Jablonowski Society announces the following prize problem for 1924: In extending the Gaussian algorithm of the arithmetic-geometric mean to the case of two arbitrary functions $\varphi(x, y)$ and $\psi(x, y)$ two infinite sequences $x, x_1, \cdots, x_n, \cdots, y, y_1, \cdots, y_n, \cdots$ are obtained, starting from any two numbers x and y, by means of the recurrence formulas $x_n = \varphi(x_{n-1}, y_{n-1})$, $y_n = \psi(x_{n-1}, y_{n-1})$. The Society desires a discussion of such sequences for some simple functions $\varphi(x, y)$ and $\psi(x, y)$ for which an independent representation of the members of the sequences defined by the recurrence formulas can be found. Competing memoirs should be sent to the Jablonowski Society at the Library of the University of Leipzig before October 31, 1924; they should be written in German, Latin, or French.

Professor Henri Lebesgue, of the Sorbonne, has been elected a member of the Paris Academy of Sciences in the section of geometry, as successor to the late Camille Jordan.

Princeton University has conferred the honorary degree of doctor of science on Professor A. G. Webster, of Clark University.

Mr. W. W. Rouse Ball, of Trinity College, Cambridge, has offered to the University a sum of £500 to constitute a trust fund for the provision of occasional lectures dealing either with some particular development of mathematics or with some application of mathematics to science.

The Royal Society of Edinburgh has conferred its Keith prize (1919–1921) on Professor R. A. Sampson for his astronomical researches, including his papers on *Studies in clocks and time keeping*, published in its PROCEEDINGS during the period of the award. The same Society has conferred its James Scott prize, established in 1918 for a lecture or essay on the fundamental concepts of natural philosophy, on Professor A. N. Whitehead, for his lecture entitled *The relatedness of nature*.

The British commissioners of 1851 announce the following appointments to senior studentships in mathematics for 1922: Mr. A. E. Ingham, research student, Cambridge University; Mr. J. E. Jones, lecturer, University of Manchester. Holders of these studentships are given the opportunity of devoting their whole time for a period of not less than two years to scientific research.

The following advanced courses in mathematics are announced for the academic year 1922–23:

UNIVERSITY OF CHICAGO.—All courses meet four times a week for a quarter of twelve weeks. Courses which continue for more than one quarter are indicated with Roman numerals, as I, II, III, or IV.—By

Professor E. H. Moore: Vectors, matrices, and quaternions; Matrices in general analysis I, II, III, IV.—By Professor L. E. Dickson: Theory of numbers I, II; Solid analytics; Theory of equations.—By Professor H. E. Slaught: Differential equations; Elliptic integrals; Calculus I.—By Professor G. A. Bliss: Definite integrals; Elliptic functions; Calculus II, III.—By Professor E. J. Wilczynski: Projective differential geometry I, II; Functions of a complex variable; Calculus I, II.—By Professor F. R. Moulton: Analytic differential equations I, II, III; Advanced ballistics I, II, III.—By Professor W. D. MacMillan: Analytic mechanics I, II, III; Celestial mechanics.—By Professor A. C. Lunn: Units and dimensions; Dynamics of continuous media; Canonical equations and quantum theory; Thermodynamics.—By Dr. Mayme I. Logsdon: Theory of algebraic invariants; Calculus I, II, III.—By Professor J. W. A. Young: Limits and series. Courses in research are also offered by Professor Moore in Foundations of mathematics and in General analysis, by Professor Bliss in Analysis, by Professor Dickson in Algebra and Theory of numbers, by Professor Wilczynski in Geometry, and by Professor Lunn in Applied mathematics.

University of Illinois.—By Professor E. J. Townsend: Real variables.—By Professor G. A. Miller: Finite groups.—By Professor J. B. Shaw: Linear operators.—By Professor A. B. Coble: Differential geometry.—By Professor R. D. Carmichael: Linear differential equations in real variables.—By Professor A. Emch: Automorphic functions.—By Professor A. R. Crathorne: Theory of statistics.—By Professor A. J. Kempner: Modern algebra.—By Professor H. Blumberg: Introduction to higher mathematics.

Johns Hopkins University.—By Professor F. Morley: Higher geometry (first term); Theory of functions (second term).—By Professor A. Cohen: Applications of calculus, differential equations, and mechanics.—By Professor L. S. Hulburt: Advanced calculus; Projective geometry and higher plane curves.—By Dr. J. R. Musselman: Elementary theory of probability.—Professor Morley will conduct a seminary and a reading class.

Massachusetts Institute of Technology.—By Professor F. S. Woods: Advanced calculus.—By Professor C. L. E. Moore: Theoretical aeronautics.—By Professor H. B. Phillips: Thermodynamics.—By Professor J. Lipka: Analytical mechanics.—By Dr. N. Wiener: Fourier's series and integral equations.—By Dr. George Rutledge: Theory of functions.—By Dr. S. D. Zeldin: Vector analysis.—By Dr. J. S. Taylor: Mathematics of investment.

University of Michigan.—By Professor J. L. Markley: Solid analytic geometry (first term); Theory of functions of a complex variable; Theory of functions of real variables.—By Professor J. W. Glover: Theory of probability (first term); Finite differences (second term); Advanced mathematical theory of interest and life contingencies.—By Professor W. B. Ford: Advanced calculus, with especial reference to Fourier series and harmonic analysis; Infinite series and products; Elements of the calculus of variations (first term).—By Professor L. C. Karpinski: Higher algebra; Theory of numbers; History of mathematics.—By Professor

J. W. Bradshaw: Introduction to modern geometry (second term); Projective geometry.—By Professor R. B. Robbins: Casualty actuarial theory.—By Mr. R. W. Barnard: Differential equations (first term); Mathematical theory of statistics, advanced course.—By Professor A. Ziwet: Hydrodynamics.—By Professor P. Field: Projective geometry for engineers (first term); Vector analysis (second term).—By Professor T. R. Running: Graphical methods (first term); Empirical formulas (second term); Advanced calculus (first term).—By Professor T. E. Hildebrandt: Theory of the potential (first term).—By Professor V. C. Poor: Theoretical mechanics.—By Professor L. J. Rouse: Fourier series (second term).

UNIVERSITY OF PENNSYLVANIA.—By Professor E. S. Crawley: Modern analytic geometry (first term); Differential equations (first term); Higher plane curves (second term).—By Professor G. H. Hallett: Infinite series and products (first term); Functions of a complex variable (second term).—By Professor H. B. Evans: Quaternions and vector methods (second term).—By Professor O. E. Glenn: Calculus of variations.—By Professor F. H. Safford: Mathematical theory of elasticity.—By Professor G. G. Chambers: Introduction to higher algebra.—By Professor H. H. Mitchell: Linear groups (first term); Advanced calculus (second term).—By Professor M. J. Babb: History of mathematics.—By Professor F. W. Beal: Differential geometry.—By Professor J. R. Kline: Foundations of mathematics (first term); Continuous transformations (second term).

UNIVERSITY OF WISCONSIN.—By Professor E. P. Lane: Modern analytical geometry.—By Professor E. B. Van Vleck: Functions of a real variable; Integral equations.—By Professor H. W. March: Theoretical hydrodynamics.—By Professor C. S. Slichter: Potential theory.—By Professor E. B. Skinner: Higher algebra.—By Professor A. Dresden: Calculus of variations.

YALE UNIVERSITY.—By Professor E. W. Brown: Mechanics; Advanced mechanics; Hydromechanics.—By Professor J. Pierpont: Functions of a complex variable; Projective and differential geometry; Approximation methods.—By Professor P. F. Smith: Differential equations.—By Professor W. A. Wilson: Theory of aggregates.—By Professor E. J. Miles: Advanced calculus; Calculus of variations.—By Professor J. I. Tracey: Advanced analytic geometry.—By Professor W. L. Crum: Mathematical statistics.—By Professor J. K. Whittemore: Advanced differential geometry.

Professor A. Sommerfeld, professor of mathematical physics at the University of Munich, will be in residence at the University of Wisconsin for the first semester of the coming academic year. He will hold during that period the Karl Schurz Memorial Professorship, which was established at the University of Wisconsin in 1910, and is resumed with this appointment after the interruption caused by the war. Professor Sommerfeld is expected to offer a three-hour course on Atomic structure, and a three-hour course either on the Analysis of wave propagation, or on the General theory of relativity. Professor Sommerfeld is known to American scholars in pure mathematics as well as to those in mathematical physics.

Professor L. Tonelli, of the University of Parma, has been appointed professor of higher analysis at the University of Bologna.

Professor L. Silla, of the University of Cagliari, has been appointed professor of rational mechanics at the University of Genoa.

Professor E. Bertini, of the University of Pisa, has retired from active service.

At the University of Lille, Professor A. Chatelet has been transferred from the professorship of general mathematics to that of rational mechanics; Dr. Gambier succeeds him as professor of general mathematics.

At Cambridge University, Mr. E. A. Milne, of Trinity College, has been appointed University lecturer in astrophysics, and Mr. S. Lees, of St. John's College, University lecturer in thermodynamics.

Professor W. R. Burwell, of the University of Tennessee, has been appointed assistant professor of mathematics and dean of freshmen at Brown University.

Professor K. D. Swartzel, formerly of the University of Ohio, has been appointed head of the department of mathematics at the University of Pittsburgh.

Miss Helen Barton, of Wellesley College, has been appointed dean of women at Albion College.

Mr. W. H. Hill, of the University of Colorado, has been appointed assistant professor of mathematics at the Manual Training Normal School, Pittsburg, Kansas.

Mr. H. K. Cummings has resigned his instructorship in mathematics at Brown University to accept a position in the Bureau of Standards at Washington.

At Iowa State College, Associate Professor J. T. Colpitts has been granted leave of absence for the coming year to study at Cornell University; Assistant Professor E. C. Kiefer has been granted leave of absence for the coming year to study at the University of Michigan; Dr. H. C. Gossard, formerly of the University of Oklahoma and the U. S. Naval Academy, has been appointed to an assistant professorship.

Professor W. J. Risley has resigned as professor of mathematics at James Millikan University, where he has been since 1910, to accept a position as head of the department of mathematics at the Colorado School of Mines.

Professor William Marshall, who has been acting head of the department of mathematics at Purdue University during the past year, has been appointed head of the department there.

At the California Institute of Technology, Professor Harry Bateman, formerly professor of aeronautical research and mathematical physics, has been made professor of mathematics, theoretical physics, and aeronautics; Dr. Clyde Wolfe, formerly instructor, has been made assistant professor of mathematics; Dr. R. C. Tolman has been appointed professor of physical chemistry and mathematical physics.

At the University of Manitoba, Mr. F. S. Nowlan has been promoted to an assistant professorship of mathematics.

Professor J. D. Bond, of Louisiana State University, has been appointed associate professor of mathematics at the University of Tennessee.

At the University of Colorado, Assistant Professor G. H. Light has been promoted to a full professorship and Dr. Claribel Kendall to an assistant professorship.

At the University of Washington, Associate Professor E. T. Bell has been promoted to a full professorship and Dr. L. L. Smail to an assistant professorship. Dr. Hermance Mullemeister has been granted a year's leave of absence for study in Holland; her place will be taken by Miss E. D. Pepper.

Assistant Professor R. B. Robbins will return to the University of Michigan after two years' absence in actuarial work in the departments of insurance of Missouri and New York.

At the University of California, Professor C. A. Noble has been granted a term's leave of absence, which he will spend in Europe.

At the Georgia School of Technology, Professor B. M. Boerckel has resigned and Mr. R. M. Mundorff has been appointed assistant professor.

At Dartmouth College, Assistant Professor F. M. Morgan has resigned.

The following appointments to instructorships at American colleges and universities are announced:

Brown University, Mr. J. H. Fithian, Mr. H. C. Hicks, Mr. A. O. Hickson;

University of California, Dr. P. H. Daus (transferred to southern branch), Dr. B. C. Wong;

Carnegie Institute of Technology, Mr. George Parks;
Dartmouth College, Dr. B. H. Brown;
Georgia School of Technology, Mr. H. K. Fuhner;
Hood College, Miss M. C. Packer;
University of Iowa, Mr. R. E. Kennon;
Iowa State College, Mr. R. G. Robinson;
University of Michigan, Mr. R. V. Churchill, Mr. C. C. Craig;
University of Missouri, Mr. C. G. Jaeger;
University of Nebraska, Mr. F. S. Harper;
New York University, Mr. F. W. John;
University of Pennsylvania, Mr. J. M. Thomas;
Pennsylvania State College, Mr. R. H. Marquis;
Purdue University, Dr. W. E. Edington, Mr. W. J. Wagner;
Randolph-Macon College, Mr. J. W. Blincoe;
Smith College, Miss Constance Wiener;
Syracuse University, Miss May Sperry;
Wellesley College, Miss F. M. Merriam;
University of Wisconsin, Mr. E. B. Keller, Mr. A. H. Wait.

Professor O. Tedone, of the University of Genoa, died April 18, 1922, at the age of fifty-two years.

Professor Hermann Grassmann, of the University of Giessen, died in January, 1922, at the age of sixty-five years.

Professor J. C. Kapteyn, professor of astronomy and mechanics at the University of Groningen, died June 18, 1922.

Professor Pierre Boutroux, of the Collège de France, died June 25, 1922. Professor Boutroux had been a member of the American Mathematical Society since 1913.

Professor James McMahon, of Cornell University, died June 1, 1922, at the age of sixty-six years. Professor McMahon had been a member of the American Mathematical Society since its organization (as the New York Mathematical Society) in 1891. He was for seven years associate editor of the Annals of Mathematics, and was secretary of section A, general secretary, and vice-president of the American Association for the Advancement of Science.

NEW PUBLICATIONS

I. HIGHER MATHEMATICS

BENNY (L. B.). Plane geometry. An account of the more elementary properties of the conic sections, treated by the methods of co-ordinate geometry, and of modern projective geometry, with applications to practical drawing. London, Blackie, 1922. 8 + 336 pp.

BIEBERBACH (L.). Funktionentheorie. (Teubners Technische Leitfäden, Band 14.) Leipzig, Teubner, 1922. 118 pp.

BOREL (E.). See LÉVY (P.).

CARTAN (E.). Leçons sur les invariants intégraux. Paris, Hermann, 1922. 10 + 210 pp.

DENJOY (A.). Calcul des coefficients de la série trigonométrique convergente la plus générale dont la somme est une fonction donnée. Paris, Gauthier-Villars, 1921. 4to. 16 pp.

GOURSAT (E.). Leçons sur le problème de Pfaff. Paris, Hermann, 1922. 8 + 387 pp.

HAAG (J.). Cours complet de mathématiques spéciales. Tome 2: Géométrie. Paris, Gauthier-Villars, 1921. 8vo. 6 + 662 pp.

HADAMARD (J.). See JUVET (G.), LÉVY (P.).

JUVET (G.). Introduction au calcul tensorial et au calcul différentiel absolu. Préface de M. J. Hadamard. Paris, Blanchard, 1922. 8vo. 2 + 101 pp.

KOWALEWSKI (G.). Die klassischen Probleme der Analysis des Unendlichen. Ein Lehr- und Uebungsbuch für Studierende zur Einführung in die Infinitesimalrechnung. 2te Auflage. Leipzig, Engelmann, 1921. 8vo. 8 + 342 pp.

LÉVY (P.). Leçons d'analyse fonctionnelle. Avec une préface de M. J. Hadamard. (Collection de Monographies sur la Théorie des fonctions, publiée sous la direction de M. E. Borel.) Paris, Gauthier-Villars, 1922. 6 + 442 pp.

LINDOW (M.). Differentialgleichungen. (Aus Natur und Geisteswelt, Nr. 589.) Leipzig, 1921.

POST (J. F.). Ueber die Darstellung ganzer Zahlen als Summen von sieben Kuben. (Diss., Halle-Wittenberg.) Halle, Hohmann, 1920. 36 pp.

RIEHLE (A.). Ueber den Bertinischen Satz und seine Erweiterung. (Diss.) Tübingen, 1920.

SIMON (P.). La recherche des lieux géométriques en géométrie analytique. Paris, Colin, 1922. 8vo. 232 pp.

UHLER (A.). Sur les séries zétafuchsiennes. (Diss., Lund.) Lund, 1921. 57 pp.

II. ELEMENTARY MATHEMATICS

AUGUST (E. F.). Vollständige logarithmische und trigonometrische Tafeln. Berlin, 1922.

BARTHOLOMEW (W. E.). See EDGERTON (E. I.).

EDGERTON (E. I.) and BARTHOLOMEW (W. E.). Business mathematics. New York, Ronald Press Company, 1921. 6 + 305 pp.

FORD (W. B.). A brief course in college algebra. New York, Macmillan, 1922.

KEAL (H. M.) and LEONARD (C. J.). Mathematics for shop and drawing students. New York, Wiley, 1921.

LEONARD (C. J.). See KEAL (H. M.).

PASSANO (L. M.). Calculus and graphs simplified for a first brief course. New York, Macmillan, 1921. 8 + 167 pp.

SICELOFF (L. P.), WENTWORTH (G.) and SMITH (D. E.). Analytic geometry. Brief course. Boston, Ginn, 1922. 6 + 186 pp. $1.80

SMITH (D. E.). See SICELOFF (L. P.).

WENTWORTH (G.). See SICELOFF (L. P.).

III. APPLIED MATHEMATICS

BACH (C.). Elastizität und Festigkeit. 8te, vermehrte Auflage. Unter Mitwirkung von R. Baumann. Berlin, Springer, 1920.

BAUMANN (R.). See BACH (C.).

BAVINK (B.). Grundriss der neueren Atomistik. Leipzig, Hirzel, 1922. 8vo. 6 + 130 pp.

BELLENOT (H.). See BORN (M.).

BLANC (A.). Rayonnement. Principes scientifiques de l'éclairage. Paris, Colin, 1921. 16mo. 212 pp.

BOREL (E.). L'espace et le temps. Paris, Alcan, 1922. 16mo. 245 pp.

BORN (M.). La constitution de la matière. Traduit par H. Bellenot. Paris, Blanchard, 1922. 8vo. 84 pp.

BROMWICH (T. J. I'a.). Examples in optics. Cambridge, Bowes and Bowes, 1921. 16 pp.

CASE (J.). Notes and examples on the theory of heat and heat engines. Cambridge, Heffer, 1922.

CLARKE (H. T.). See PLANCK (M.).

CLIBBENS (D. A.). The principles of the phase theory. Heterogeneous equilibria between salts and their aqueous solutions. London, Macmillan, 1920. 20 + 382 pp. 25s.

COWLEY (W. L.) and LEVY (H.). Aeronautics in theory and experiment. 2d edition. London, Arnold, 1920. 12 + 331 pp. 25s.

DUBROCA (M.). Au sujet de la théorie de la relativité restreinte. 1e partie. Dijon, Imprimerie R. de Thorey, 1921. 8vo. 76 pp.

EMSLEY (H. H.). See GLEICHEN (A.).

FÖPPL (A.). Vorlesungen über technische Mechanik. Band 1: Einführung in die Mechanik. 7te Auflage. Band 2: Graphische Statik. 5te Auflage. Band 3: Festigkeitslehre. 8te Auflage. Band 4: Dynamik. 6te Auflage. Leipzig, 1920–1921.

GEIGER (H.) und MAKOWER (W.). Messmethoden auf dem Gebiete der Radioaktivität. Braunschweig, Vieweg, 1920. 9 + 156 pp.

GLEICHEN (A.). The theory of modern optical instruments. Translated from the German by H. H. Emsley and W. Swaine. 2d edition. London, H. M. Stationery Office, 1921. 11 + 365 pp.

GLOVER (J. W.). See UNITED STATES.

GRAMMEL (R.). Die mechanischen Beweise für die Bewegung der Erde. Berlin, Springer, 1922. 2 + 71 pp.

HALDANE (VISCOUNT). La règne de la relativité. Traduction française de H. de Varigny. Paris, Gauthier-Villars, 1922. 8vo. 590 pp.

HIGGINS (A. L.). The transition spiral and its introduction to railway curves. With field exercises in construction and alignment. London, Constable, 1921. 8 + 111 pp. 6s.

VON IGNATOWSKY (W.). Die Vektoranalysis und ihre Anwendung in die theoretischen Physik. 1ter Teil. 2te Auflage. Leipzig, 1921.

JUVET (G.). See WEYL (H.).

LAURENT (M.). Les trois états de matière. Etat fluide, état corpusculaire, corps simples. Paris, Maloine, 1922. 8vo. 52 pp.

LECORNU (L.). Dynamique appliquée. 2e édition, revue, corrigée et augmentée. 2 volumes. Paris, Doin, 1921.

LENARD (P.). Ueber Relativitätsprinzip. Aether, Gravitation. Neue, vermehrte Ausgabe. Leipzig, Hirzel, 1920.

LEROY (R.). See WEYL (H.).

LEVY (H.). See COWLEY (W. L.).

DE LOSADA Y PUGA (C.). Las anomalias de la gravidad. Lima, Torres Aguirre, 1920.

LOTZE (A.). Die Grundgleichungen der Mechanik insbesondere starrer Körper, neu entwickelt mit Grassmanns Punktrechnung. Leipzig, Teubner, 1922. 8vo. 50 pp.

MCCABE (J.). See NORDMANN (C.).

MAGGI (G. A.). Elementi di statica e teoria dei vettori applicati. Pisa, Spoerri, 1920. 8vo. 176 pp.

MAKOWER (W.). See GEIGER (H.).

MIALL (S.). The structure of the atom. Notes on some recent theories. London, Benn, 1922. 3 + 26 pp.

MILLS (J.). Within the atom. Popular view of elektrons and quanta. New York, Van Nostrand, 1921. 232 pp. $2.25

NORDMANN (C.). Einstein and the universe. A popular exposition of the famous theory. Translated by J. McCabe. London, Fisher Unwin, 1922. 185 pp.

OPPENHEIM (S.). Das astronomische Weltbild im Wandel der Zeit. I: Vom Altertum bis zur Neuzeit. 3te Auflage. Leipzig, Teubner, 1920.

PHILIP (A.). The calendar. Its history, structure, and improvement. Cambridge, University Press, 1921. 12 + 104 pp. 7s. 6d.

PILLET (J.). Traité de géométrie descriptive. Nouveau tirage. Paris, 1921. 4to. 270 pp.

PLANCK (M.). Einführung in die Theorie der Elektrizität. Leipzig, 1922.

——. The origin and development of the quantum theory. Translated by H. T. Clarke and L. Silberstein. Oxford, Clarendon Press, 1922. 8vo. 23 pp.

READE (W. H. V.). A criticism of Einstein and his problem. Oxford, Blackwell, 1922. 6 + 126 pp.

VON ROHR (M.). Theorie der optischen Instrumente. Berlin, 1922.

RUSSELL (R. A. B.). See THIRRING (J. H.).

SCHMIDT (H.). Das Weltbild der Relativitätstheorie. 2te, vermehrte Auflage. Hamburg, 1920.

SILBERSTEIN (L.). See PLANCK (M.).

SOMMERFELD (A.). Atombau und Spektrallinien. 3te Auflage. Braunschweig, Vieweg, 1922. 8vo.

SWAINE (W.). See GLEICHEN (A.).

TAMMANN (G.). Aggregatzustände. Die Zuständigkeitsveränderungen der Materie von Druck und Temperatur. Leipzig, Voss, 1922. 8 + 294 pp.

THIRRING (J. H.). The ideas of Einstein's theory. A theory of relativity in simple language. Translated by R. A. B. Russell. London, Methuen, 1921. 15 + 167 pp. 5s.

TIMERDING (H. E.). Die Fallgesetze. 2te Auflage. Leipzig, Teubner, 1921. 4 + 51 pp.

UNITED STATES life tables, 1890, 1901, 1910, and 1901–1910. Explanatory text, mathematical theory, computations, graphs, and original statistics, also tables of United States life annuities, life tables of foreign countries, mortality tables of life insurance companies. Prepared by J. W. Glover. Washington, Government Printing Office, 1921. 4to. 496 pp.

DE VARIGNY (H.). See HALDANE (VISCOUNT).

VELASCO DE PANDO (M.). Calcolo de las probabilides. Paris, Gauthier-Villars, 1920. 8vo. 430 pp.

VILLEY (J.). Physique élémentaire et théories modernes. I: Molécules et atomes. Paris, Gauthier-Villars, 1921. 8vo. 10 + 198 pp.

WEYL (H.). Temps, espace, matière. Leçons sur la théorie de la relativité générale. Traduit sur la 4e édition allemande par G. Juvet et R. Leroy. Paris, Blanchard, 1922. 8vo. 8 + 290 pp.